

In the following *Annual Report 2013*, I refer to the working plan [1, § 1.6, pp. 16–17] of my project proposal, [1] *Towers of  $p$ -class fields over algebraic number fields*, in the file `Proposal20130213.pdf`. I also refer to certain URLs in the world wide web, in particular, to my *Website for Principal Investigators of the FWF*, [2] <http://www.algebra.at/FWFP26008N25/index.html>. In the initial stage of my project, from July to December 2013, I focussed on further development and *international communication* of my outstanding discoveries, providing proven evidence for the first  $p$ -class field towers of length bigger than two, for an odd prime  $p$ , namely  $p = 3$  [1, p. 16, First research line, item (1.a)]. Two coauthors were involved in these developments. Michael R. Bush in the proofs for the *ground state* of such towers, with relative group of smallest possible order 6 561, and metabelianization with transfer kernel type (TKT) E.9. Mike F. Newman in analyzing *excited states*, with relative groups of order at least 177 147, and TKTs E.6, 8, 9, 14.

1. Since there arose the possibility to meet M. R. Bush personally, I prepared a presentation *Finite 3-groups as viewed from class field theory* for the conference *Groups St Andrews 2013*, on August 9, abstract: <http://at.yorku.ca/cgi-bin/abstract/cbhp-24>, slides: [2, publ., (b.2)], conference program: <http://www.groupsstandrews.org/2013/programme.shtml>, and an illuminating poster, giving a succinct survey of the coclass project [2, publ., (b.3)]. The meeting with Bush enabled a fine tuning of details concerning our joint paper *3-class field towers of exact length 3* [2, publ., (a.2)].

2. Inspired by other presentations in St Andrews, I wrote two spontaneous purely group-theoretic preprints [2, publ., (b.1)], not planned in the proposal, *Normal lattice of certain metabelian  $p$ -groups* and *Parametrized  $p$ -presentations of periodic sequences of 3-groups*, in August. The former provides foundations for investigating TKTs in sections E and F. In the latter, I discovered a very promising characterization of mainline vertices on coclass trees which seems to be of a fairly general nature and obviously has not been recognized by other investigators up to now. *Mainline Principle: The relators for  $p$ th powers of generators of mainline groups are distinguished by being independent of the class*, whereas a relator of any non-mainline group contains the generator of the last non-trivial term of the lower central series as a *small perturbation*. This principle will be of greatest importance for the further processing of my project.

3. For the *ÖMG and DMV Congress 2013*, Innsbruck, I prepared my presentation *3-class field towers of exact length 3* on Sept. 24, abstract: <http://www.algebra.at/DCMayer0eMG2013.pdf>, slides: [2, publ., (b.2)].

4. In December, M. R. Bush and I submitted our article *3-class field towers of exact length 3* [2, publ., (a.2)] to the *Journal of Number Theory* (Elsevier), and to the open source repository arXiv. To begin with a brief and compact summary and to avoid the danger of loosing priority, we restricted our announcements on the one hand to the ground state of TKTs in section E, corresponding to Schur  $\sigma$ -groups of order  $3^8$ , and on the other hand to the special TKT E.9, which is completely sufficient to provide the first rigorous disproof of the claims by Scholz/Taussky and Heider/Schmithals. For the sake of brevity, we refrained from shedding light on Brink/Gold.

5. The most recent analysis of the results by Brink/Gold, in cooperation with Mike F. Newman, from the viewpoint of descendant trees of 3-groups, underpinned by the Mainline Principle (in 2.), was presented in my lecture *Class towers and capitulation over quadratic fields* at the *West Coast Number Theory Conference 2013* on Dec. 18, program: <http://westcoastnumbertheory.org/schedule/2013-schedule-of-talks/>, slides: [2, publ., (b.2)]. As an ultimate summary of all my international presentations in 2013 [2, publ., (b.2)] and of my tetralogy [2, publ., (a.2)], I gained insight into a new *strategy of pattern recognition* [2, publ., (b.1)] which makes use of a systematic partial order on TKTs and TTTs of vertices on descendant trees of  $p$ -groups and will be fundamental for all further developments in my project.

6. In January 2014 I started a web presentation concerning pure quintic fields [1, p. 17, Second research line, item (3)], featuring my investigations in November and December 2013 [2, further, (1.1)]. My hypothetical classification of pure quintic fields, according to three *types of principal factorizations* (absolute, intermediate, and relative) and to several possibilities for the group of *relative norms of units*, turned out to be in perfect accordance with extensive numerical computations.