APPLICATION FOR FUNDING OF PROJECT-SPECIFIC EQUIPMENT REQUIRED FOR THE STAND-ALONE PROJECT P 26008-N25, "TOWERS OF p-CLASS FIELDS OVER ALGEBRAIC NUMBER FIELDS"

DANIEL C. MAYER

1. Erroneous assumptions in the original project proposal

In the section 4 *Financial Aspects*, subsection 4.1 *Information on the Research Institution*, subsubsection *Available Infrastructure*, of my project proposal [9], I have written:

> "... Our private *office* and *laboratory* is endowed with several fast, powerful, and most up-to-date notebook, laptop, and desktop computers, either with Intel i5 quad core processor running Mac OS X Lion or with Intel i7 quad core hyperthreading CPU running MS Windows 7 Professional. All machines have (at least) 4 GB RAM, run 64 bit operating systems, and possess high speed internet connections over WLAN.
>
> For the completion of numerical aspects of our stand alone project, we have available several free open-source software packages, the Computer Algebra System *Pari/GP*, the System for Computational Discrete Algebra *GAP*, and the Computer Algebra System *Singular*. Finally, we own a three years licence of the Computational Algebra System *Magma* V2.19-3 from the Computational Algebra Group at the University of Sydney, valid from December 2011 to November 2014, to be renewed and extended at the date of expiration."

The reason for this confident attitude was my firm conviction that I were in the possession of the top models of desktop and notebook computers which are currently available at vendors like Media Markt and Saturn.

The logical consequence was my statement in the subsection 4.2 *Information on the Requested Support*, subsubsection *Justification for Non-Personnel Costs*, of the proposal [9], as follows:

> "Since our private research location is sufficiently endowed with infrastructure perfectly adequate to complete the present stand alone project ..., we do not require an additional application for further support."

2. Unexpected and unpleasant realizations

On February 13, 2013, I submitted my proposal [9] for a three years stand-alone research project with title "Towers of p-class fields over algebraic number fields" to the Austrian Science Fund (FWF). The project was granted on June 24, 2013, and received the identifier P 26008-N25. At that early stage, I was firmly convinced that the machines with numbers 1 and 2 in Table 1 will be sufficient for running my licence of the Computational Algebra System Magma [3, 4, 8] from the University of Sydney, which is essential for reaching the targets of my project.

Date: June 08, 2015.

However, on October 02, 2013, only a month after the official project start, a first memory problem arose and I wrote to Prof. John Cannon (`john@maths.usyd.edu.au`) and the Magma Administration (`admin@maths.usyd.edu.au`):

> ". . . We have been successfully using our licence "Graz-QuantumAlgebra" of Magma for two years since December 2011 without memory problems so far. The requirements of our new project, however, are so extensive that some basic preparatory Magma sessions caused a hopeless exhaustion of the entire memory available . . ."

The reason for the problems was the construction of 3-groups with transfer kernel types in section F [10] which I wanted to investigate in collaboration with Mike F. Newman. My suggestion to transfer my licence to a mainframe computer at the University of Technology or the Karl-Franzens University Graz turned out to be complicated and so I prefered to postpone the groups in section F and to wait until November 2014, the date where my licence was going to expire.

In August, 2014, the p-group generation algorithm [7] by Newman [16] and O'Brien [17], which is implemented in the ANUPQ package [6] of Magma [8] and GAP [5], gave rise to massive memory problems, when I tried to find the second 3-class groups of complex quadratic fields with 3-class groups of type $(3, 3, 3)$ by means of my recently developed strategy of pattern recognition via Artin transfers [11, 12, 15].

Again, I wrote to the Group Pub Forum, to the GAP Forum, and to the Magma Administration, using the SmallGroups identifiers [1, 2] of finite p-groups (August 10, 2014):

> ". . . Descendant calculation by means of the p-group generation algorithm reveals massive memory problems for finite 3-groups with abelianization of type $(3, 3, 3)$, for instance SmallGroup(243,37) and SmallGroup(729,122). For the former, step sizes 1,2,3 are feasible, whereas 4,5 cause failure. For the latter, only step size 1 is possible, whereas 2,. . .,8 exhaust the complete memory. But I urgently need information on descendants of larger step sizes. What should I do?"

3. Documentation of failures

On August 15, 2014, I wrote to several Web Forums:

> "Most recent STRIKING NEWS FOR all USERS OF the MAGMA computer algebra system:
>
> Due to a new problem with constructing the immediate descendants of step size 3 of the metabelian 3-group SmallGroup(2187,4487) with nuclear rank 3 (which is itself an immediate descendant of step size 1 of the critical group SmallGroup(729,122) mentioned in my last contributions) I asked for help by the Magma group in Sydney.
>
> Unfortunately, the response by Prof. John Cannon and Stephen Donnelly was surprising and unpleasant, but at least it explained the reasons for the failure completely:
>
> In the process of descendant generation, Magma computes the orbits of a matrix group acting on an underlying vector space. At present, the number of objects allowed in a set, sequence or list is limited to $2^{30} - 1 = 1\,073\,741\,823$. However, in the calculations for SmallGroup(2187,4487) a set with $3^{21} = 10\,460\,353\,203$ objects arises and so Magma aborts the process, INDEPENDENTLY OF THE amount of available RAM !!! (So it is not simply the fault of my computers.)
>
> Prof. Cannon promised that this limit will probably be raised in 2015. Nevertheless, he warns against being euphoric, since it is very common that p-group generation fails at this point and the issues concerning the needed amount of RAM will remain."

Finally, an explicit error message on my Windows machine number 1 in Table 1 revealed an annoying secret which unfortunately was not documented publicly in the Magma handbook and website. On August 17, 2014, I got the following message when I was executing the

p-group generation algorithm as implemented in Magma on a Windows 7 Pro 64 bit machine with 8 GB RAM. The fatal statements were simply:

```
$ H := SmallGroup(729,122);
$ Des := Descendants(H,NilpotencyClass(H)+1: StepSizes := [2]);
(RAM error message, Windows)
Current total memory usage: 1322.5MB, failed memory request: 180.8MB
System error: Out of memory.
All virtual memory has been exhausted so Magma cannot perform this statement.
```

On the same day, Prof. John Cannon gave the following explanation.

> "... I am sorry to say that running Magma on a Windows machine is to be avoided. Because of a very stupid decision by MS when making the change from 32 to 64-bits we only support 32-bit Windows machines. Few of our users use Windows and so far there haven't been enough users to justify the very expensive task (at least a man-year) of producing a 64-bit Windows version. So the maximum amount of memory Magma can use under Windows is about 1.4 GB no matter how much memory the machine actually has. I am sorry that I didn't notice this before. Anyway this explains this message."

So the 8GB RAM of my machine number 1 in Table 1 were completely useless with respect to executing Magma programs, since the 32 bit version can use at most 1.4GB, on principle. An annoying secret which I did not know for nearly three years from 2011 to 2014. As a conclusion, on August 17, 2014, Prof. John Cannon (the leader of the Magma Group) suggested to purchase a powerful Linux machine:

```
> Dear Dr Mayer,
> ***
> I would strongly recommend you get a linux machine with 128 GB
> given the sophistication of what you are doing.
> Magma has been tested on jobs using up to 1 TB
> and should be able to use much more memory
> though this has not been tested.
> ***
> Best wishes,
> John
```

4. Modification of the infrastructure

On October 19, 2014, Prof. Cannon provided some useful hints for selecting a machine for running the Magma system.

> "... First of all there are the multinational computer companies such as Dell, IBM, HP and some Japanese firms. Dell offers low prices but this can come at the expense of the quality of various components such as the internal power supply, disks, fans etc. IBM and HP provide better quality but they are overly expensive (esp IBM).
>
> The machine that you want is likely to be sold by local companies who assemble machines from the basic components. You probably need to find out the names of two or three such companies and then talk to them about the configuration you are seeking.
>
> When you specify your machine, you should insist that the motherboard should be based on Intel Sandy Bridge or Ivy Bridge processors. These are the ones we specify.

Recently I foolishly bought Dells for the groups' desktops. And now I regret it. For many years we used only Sun equipment but then Oracle took it over and killed off most of the computer lines. So now we look around and buy what looks best at the time. The commodification of desktop computers has resulted in that either they are cheaper than they ought to be and made to a low standard or they cost more than is reasonable. "

Grateful for Prof. Cannon's suggestions, I began getting in contact with several vendors of powerful machines: the business center of A. Händler, the IT services company DCCS, both certified IBM vendors, and the company Digitalis, a certified Apple vendor. It turned out that Lenovo is actually continuing the former work station series of IBM. The proposals of these vendors are attached in an archive file `Documents.rar` and are summarized in numbers 3 to 7 of Table 1.

Table 1. Random Access Memory and Central Processing Units

No.	Producer	Model	RAM	CPUs	Co	Th	Freq	OS
1	Toshiba	Qosmio X500-12F	8 GB	1 Intel i7	4	8	1.6 GHz	Windows 7 Pro
2	Apple	iMac 27"	4 GB	1 Intel i5	4	4	2.7 GHz	Mac OS X Lion
3	Lenovo	ThinkStation D30	256 GB	2 Intel Xeon	16	32	2.0 GHz	Ubuntu 14.04 LTS
4	Lenovo	ThinkStation P900	256 GB	2 Intel Xeon	12	24	2.4 GHz	Linux
5	Apple	Mac Pro	64 GB	1 Intel Xeon	12	24	2.7 GHz	Mac OS X
6	Apple	iMac 27" Retina	32 GB	1 Intel i7	4	8	4.0 GHz	Mac OS X Yosemite
7	Apple	MacBook Pro	16 GB	1 Intel i7	4	8	2.5 GHz	Mac OS X Yosemite

Due to a thorough comparison of prices and performance of the machines, I finally decided to purchase the new machines numbers 3, 6, and 7. Number 3 for replacing the old number 1, because it clearly outperformed numbers 4 and 5, and enabled the revolutionary results in my most recent papers and presentations [11, 12, 14, 15]. (It was necessary to install an uninterruptible power supply (UPS) for machine number 3.) Number 6 for replacing the old number 2, since the frequency 4GHz is at the upper limit of what is possible at the moment. And number 7, since I was invited to give lectures and exercises [13] at the International Research School CIMPA in Oujda Morocco from May 18 to 29, 2015, and to teach young postdoc researchers from Morocco, Algeria, Senegal and Nigeria how to use Magma for various kinds of scientific computations.

5. Support for new equipment

I would be very grateful, if the Austrian Science Fund could support at least the renewed Magma licence from December 2014 to November 2017 (maintenance and support for Linux and OS X executable images of the Magma computational algebra system) and the expenses for the central machine number 3 (the Lenovo ThinkStation D30 with two Intel Xeon 8 Core CPUs).

References

[1] H.U. Besche, B. Eick, and E.A. O'Brien, *A millennium project: constructing small groups*, Int. J. Algebra Comput. **12** (2002), 623-644.

[2] H.U. Besche, B. Eick, and E.A. O'Brien, *The SmallGroups Library — a Library of Groups of Small Order*, 2005, an accepted and refereed GAP 4 package, available also in MAGMA.

[3] W. Bosma, J. Cannon and C. Playoust, *The Magma algebra system. I. The user language*, J. Symbolic Comput. **24** (1997), 235–265.

[4] W. Bosma, J.J. Cannon, C. Fieker and A. Steels (eds.), *Handbook of Magma functions* (Edition 2.21, Univ. of Sydney, Sydney, 2015).

[5] The GAP Group, *GAP – Groups, Algorithms, and Programming — a System for Computational Discrete Algebra*, Version 4.7.7, Aachen, Braunschweig, Fort Collins, St. Andrews, 2015, (http://www.gap-system.org).

[6] G. Gamble, W. Nickel, and E.A. O'Brien, *ANU p-Quotient — p-Quotient and p-Group Generation Algorithms*, 2006, an accepted GAP 4 package, available also in MAGMA.

[7] D.F. Holt, B. Eick, and E.A. O'Brien, *Handbook of computational group theory*, Discrete mathematics and its applications, Chapman and Hall/CRC Press, Boca Raton, 2005.

[8] The MAGMA Group, *MAGMA Computational Algebra System*, Version 2.21-4, Sydney, 2015, (http://magma.maths.usyd.edu.au).

[9] D.C. Mayer, *Towers of p-class fields over algebraic number fields*, Proposal of a stand-alone project, Austrian Science Fund (FWF): P 26008-N25, Proposal20130213.pdf.

[10] D.C. Mayer, *3-groups with transfer kernel type* F, http://www.researchgate.net/profile/DanielMayer3/.

[11] D.C. Mayer, *Periodic bifurcations in descendant trees of finite p-groups*, Adv. Pure Math. **5** (2015), no. 4, 162–195, DOI 10.4236/apm.2015.54020, Special Issue on Group Theory, March 2015.

[12] D.C. Mayer, *Index-p abelianization data of p-class tower groups*, Adv. Pure Math. **5** (2015), no. 5, 286–313, DOI 10.4236/apm.2015.55029, Special Issue on Number Theory and Cryptography, April 2015.

[13] D.C. Mayer, *La Théorie Algorithmique des Nombres, Construction des Corps de Nombres Algébriques et Corps de Classes*, École de Recherche CIMPA UNESCO Maroc, Théorie des Nombres et ses Applications, Université Mohammed Premier, Faculté des Sciences (FSO), Oujda, Maroc, Mai 2015.

[14] D.C. Mayer, *Index-p abelianization data of p-class tower groups*, 29th Journées Arithmétiques, Univ. of Debrecen, Hungary, presentation to be delivered 10 July 2015.

[15] D.C. Mayer, *Periodic sequences of p-class tower groups*, to appear in J. Appl. Math. Phys. (arXiv: 1504.00851v1 [math.NT] 3 Apr 2015.) (International Conference on Groups and Algebras 2015, Shanghai, China, presentation to be delivered 21 July 2015).

[16] M.F. Newman, *Determination of groups of prime-power order*, pp. 73–84, in: Group Theory, Canberra, 1975, Lecture Notes in Math., vol. 573, Springer, Berlin, 1977.

[17] E.A. O'Brien, *The p-group generation algorithm*, J. Symbolic Comput. **9** (1990), 677–698.

Naglergasse 53, 8010 Graz, Austria
E-mail address: quantum.algebra@algebra.at
URL: http://www.algebra.at