

A Succinct Survey of the Coclass Project

Coclass 1	p -Groups	3-Groups
[Ph. Hall &] Blackburn , 1958: $\mathcal{G}(p, 1)$, p -groups of maximal class, a single coclass tree $\mathcal{T}_1(C_p \times C_p)$, entirely metabelian for $p \in \{2, 3\}$	Leedham-Green, Newman , 1975: digraph of all finite p -groups $\mathcal{G}(p) = \cup_{r=0}^{\infty} \mathcal{G}(p, r)$, partitioned into coclass graphs	[Newman &] Ascione , 1979: $\mathcal{G}(3, 2)$, $G/G' \simeq (3, 3)$ or $(3, 9)$, 3-groups of second maximal class, 15, resp. 16, coclass trees
Miech , 1970: $\mathcal{M}(p, 1) \subset \mathcal{G}(p, 1)$, $p \geq 5$, metabelian p -groups of maximal class	Shalev , 1995, and Leedham-Green , 1995: $\mathcal{G}(p, r) = \mathcal{G}_0(p, r) \cup [\cup_i \mathcal{T}_r(G_0^{(i)})]$ finitely many sporadic groups and finitely many coclass trees	[Jehne &] Nebelung , 1989: metabelian 3-groups G with $G/G' \simeq (3, 3)$, $\tilde{\mathcal{M}}(3) \subset \mathcal{G}(3)$, periodicity of metabelian coclass trees $\tilde{\mathcal{T}}_{r+2}(G_0^{(r+2)}) \simeq \tilde{\mathcal{T}}_r(G_0^{(r)})$, for $r \geq 3$
[Eick &] Dietrich , 2010: $\mathcal{G}(5, 1)$, 5-groups of maximal class, periodic structures, bounded width, unbounded depth	du Sautoy , 2001, and Eick, Leedham-Green , 2008: periodicity of branches of depth-pruned coclass trees $\mathcal{T}_{r,d}(G_0)$, $\mathcal{B}_d(j + \ell) \simeq \mathcal{B}_d(j)$, for $j \geq j_0$, isomorphic as graphs, for some $\ell \mid p^{r+1}(p - 1)$	Mayer, Newman , 2013: periodicity of TKT-pruned coclass trees with $\varkappa(G)$ in sections c and E, $\mathcal{T}_r^*(G_0^{(r)}) \simeq \mathcal{T}_2^*(G_0^{(2)})$, for $r \geq 3$, isomorphic as graphs