

Annual Report 2014

In the following *Annual Report 2014*, I refer to § 1, *Scientific Aspects* [1, pp.2–18], of my project proposal, *Towers of p -class fields over algebraic number fields* [1], in the file `Proposal20130213.pdf`. I also refer to the URL <http://www.algebra.at/FWFP26008N25/index.html> of my *Website for Principal Investigators of the FWF* [2]. According to [1, pp.5–6, §§ 1.3.1–1.3.2], the project consists of two research lines (RL), *Two- and Three-stage towers of unramified Hilbert p -class fields*, briefly (RL1), and *Single-stage towers of ramified p -ring class fields*, briefly (RL2). Let me begin with a target-performance comparison, based on the *Working Plan and Time Schedule* [1, pp.16–17, § 1.6]. Target [1, p.16, (RL1), item (1.a)] was completed in the proposed 2 months by publishing [3] Bush, Mayer, *3-class field towers of exact length 3*, *J. Number Theory* **147** (2015), 766–777, DOI 10.1016/j.jnt.2014.08.010. (arXiv: 1312.0251v1 [math.NT] 1 Dec 2013.) An essential supplement was added by myself alone in [9, § 21.2] (see below). In further 3 months, due to several fortunate circumstances, I unexpectedly completed alone what can be done currently for the TKT in section H [10, §§ 6.2.2–6.2.3] (see below), instead of the TKTs in section F, whose investigation has been begun in cooperation with Newman but still needs to be completed. Target [1, p.16, (RL1), item (1.b)] was completed in 3 months with results published in [10, § 6.2.1] (see below). For the target [1, p.17, (RL1), item (2.a)], the theory is now, after 2 months, completely developed with Azizi, Talbi, Derhem. Documentation is still under construction. I have communicated the numerical results with illuminating comments as a series of sequences A250236–A250242 in [4] Sloane, *The On-Line Encyclopedia of Integer Sequences* (OEIS), The OEIS Foundation Inc., 2014, (<http://oeis.org/>). See [2, Project Publications, (d.2.2)]. Target [1, p.17, (RL1), item (2.b)] was completed in 4 months and published in [5] Azizi, Zekhnini, Taous, Mayer, *Principalization of 2-class groups of type $(2, 2, 2)$ of biquadratic fields $k = \mathbb{Q}(\sqrt{p_1 p_2 q}, \sqrt{-1})$* , *Int. J. Number Theory* (2015), DOI 10.1142/S1793042115500645. For target [1, p.17, (RL2), item (1)], though the documentation is still in progress, the theory was completely developed in 2 months, based on [6] Mayer, *Quadratic p -ring spaces for counting dihedral fields*, *Int. J. Number Theory* **10** (2014), no. 8, 2205–2242, DOI 10.1142/S1793042114500754. For target [1, p.17, (RL2), item (3)], theory, experiments, data collection and evaluation were done in 3 months, documentation being under construction. The sum of 18 months exactly corresponds to the current project state. A few words are due to my new *Strategies of Dissemination* [1, p.17, § 1.7]. The last two parts of my mentioned *Tetralogy* appeared in reverse order, due to considerable delays in the refereeing process. The fourth part, submitted Dec. 2011, [7] Mayer, *The distribution of second p -class groups on coclass graphs*, *J. Théor. Nombres Bordeaux* **25** (2013), no. 2, 401–456, DOI 10.5802/jtnb842. (27th Journées Arithmétiques 2011, Vilnius University, Lithuania.) was published more than a year before the third part, submitted Aug. 2011, [8] Mayer, *Principalization algorithm via class group structure*, *J. Théor. Nombres Bordeaux* **26** (2014), no. 2, 415–464. Therefore, I decided not to use the *Journal de Théorie des Nombres de Bordeaux* (JTNB) within the frame of the *Centre de Diffusion des Revues Académiques de Mathématiques* (CEDRAM) any longer. In 2014, I conquered the *English Wikipedia*, which requires its own Wiki Markup Language, by instantly publishing four fundamental articles, (1) *Descendant tree (group theory)*, (2) *Artin transfer (group theory)*, (3) *Principalization (algebra)* and (4) *p -Group generation algorithm* [2, Project Publications, (d.1)], whose page view statistics prove that they have gained incredible popularity with an average of more than ten views per day and article. To transform the Wikipedia articles into three printed publications, I have selected the journal *Advances in Pure Mathematics* (APM) issued by the vanity press company *Scientific Research Publishing* (SCIRP). The first part of my *New Trilogy* [2, Project Publications, (a.2)] is [9] Mayer, *Periodic bifurcations in descendant trees of finite p -groups*, *Adv. Pure Math.*, vol. **5**, no. 4, Special Issue on Group Theory, March 2015. (arXiv: 1502.03390v1 [math.GT] 11 Feb 2015.) It contains the Wikipedia articles (1) and (4) as its foundation and striking news on periodic bifurcations [9, § 21.1], inspired by refereeing a paper by Azizi, Zekhnini, Taous, *Coclass of $\text{Gal}(k_2^{(2)}|k)$ for some fields $k = \mathbb{Q}(\sqrt{p_1 p_2 q}, \sqrt{-1})$ with 2-class groups of type $(2, 2, 2)$* , *J. Algebra Appl.*, 2015. The second part is [10] Mayer, *Index- p abelianization data of p -class tower groups*, *Adv. Pure Math.*, Special Issue on Number Theory and Cryptography, April 2015. (arXiv: 1502.03388v1 [math.NT] 11 Feb 2015.) It was inspired by the 2nd version of Boston, Bush, Hajir, *Heuristics for p -class towers of imaginary quadratic fields*, *Math. Annalen*, 2015. (arXiv: 1111.4679v2 [math.NT] 10 Dec 2014.) It solves problems posed in Bartholdi, Bush, *Maximal unramified 3-extensions of imaginary quadratic fields and $\text{SL}_2\mathbb{Z}_3$* , *J. Number Theory* **124** (2007), 159–166. The third part is under construction and will contain the Wikipedia article (2).