

The following *Annual Report 2015* concerns my stand-alone project *Towers of  $p$ -class fields over algebraic number fields* as described in the file `Proposal20130213.pdf`. I refer to various URLs in the world wide web, in particular, <http://www.algebra.at/FWFP26008N25/index.html>, my *Website for Principal Investigators of the FWF*. In contrast to 2014, which was free of any presentations and permitted complete concentration on preparatory work for the project objectives, 2015 was full of international events where I presented the project results. 2015 yielded the harvest of 2014 in form of publications. The first paper [1] contains foundations on descendant trees and the  $p$ -group generation algorithm, required for understanding new discoveries of periodic bifurcations with applications to covers of metabelian  $p$ -groups and identifying  $p$ -class towers. My two weeks at the CIMPA Research School in Morocco, <http://www.cimpa-icpam.org/ecoles-de-recherche/anciens-programmes/ecoles-de-recherche-2015/liste-chronologique-des-ecoles-de/article/theorie-des-nombres-et?lang=fr>, enabled fine tuning and finishing our cooperation on [9], which will appear in IJNT. The article [2] was inspired by marvellous computational results of Bush in 2014, lays the precise basis of the concept of IPADs of first and second order, and was presented at the 29th JA, Debrecen, <http://ja2015.math.unideb.hu>. During 2015, the joint papers [7] in JNT and [8] in IJNT eventually appeared in printed form. The article [3] applies group theoretic results in [1] to number theoretic problems, sheds more light on [7] and [8], and was presented in my invited talk at the 1st ICGA, Shanghai, <http://www.engii.org/ws/Home.aspx?ID=624>. Preparation and evaluation of my talk at the 22nd CSICNT, Liptovsky Jan, <http://ntc.osu.cz/2015>, required 2 months. Unexpectedly, I proved that real quadratic fields with capitulation types in section c have 3-stage towers. All results appeared in the conference proceedings [5] in TMMP, which deserves particular attention for the following reason. The main theorem in the paper by Shafarevich, which is most important for my project, contradicted our joint results in [8], resp. [9], on biquadratic fields containing fourth, resp. third, roots of unity and caused a lot of confusion until I discovered a fatal misprint in both, the russian original and the english translation, and published corrections in [5] and [9]. As announced, my *New Trilogy*, which started with [1] and [2], was completed with [4] in 2 months. The article contains a detailed theory of Artin transfers and the long desired proof of the compatibility of Artin patterns with edges of descendant trees, which thereby are endowed with additional structure. The papers [1] and [4] are devoted to pure group theory, whereas all other papers contain applications to class field theory. The last paper [6] of 2015 is a supplement of [2], containing deeper details of computational results in a more systematic way, which were given in [2] as lurid headlines only. Although they are not joint work, the papers [2], [3], [5] and [6] would have been impossible, firstly, without Bush's permission to use his extensive numerical results of 2014 and 2015, and secondly, without my new Linux workstation with Xeon processors, supported by the Austrian Science Fund. Further 2 months were required for computing the capitulation of all real quadratic fields with discriminants  $0 < d < 5 \cdot 10^7$ , which became possible by MAGMA version V2.21-8 (Nov. 2015), without a permanent bug up to V2.21-7.

[1] D.C. Mayer, *Periodic bifurcations in descendant trees of finite  $p$ -groups*, Advances in Pure Mathematics **5** (2015), no.4, 162 – 195, Special Issue on Group Theory, DOI 10.4236/apm.2015.54020.

[2] D.C. Mayer, *Index- $p$  abelianization data of  $p$ -class tower groups*, Advances in Pure Mathematics **5** (2015), no.5, 286 – 313, Special Issue on Number Theory and Cryptography, DOI 10.4236/apm.2015.55029.

[3] D.C. Mayer, *Periodic sequences of  $p$ -class tower groups*, Journal of Applied Mathematics and Physics **3** (2015), no.7, 746 – 756, DOI 10.4236/jamp.2015.37090.

[4] D.C. Mayer, *Artin transfer patterns on descendant trees of finite  $p$ -groups*, Advances in Pure Mathematics **6** (2016), no.2, 66 – 104, Special Issue on Group Theory Research, DOI 10.4236/apm.2016.62008.

[5] D.C. Mayer, *New number fields with known  $p$ -class tower*, Tatra Mountains Mathematical Publications **64** (2015), 21 – 57, Number Theory and Cryptology '15, DOI 10.1515/tmmp-2015-0040.

[6] D.C. Mayer, *Index- $p$  abelianization data of  $p$ -class tower groups, II*, J. Théor. Nombres Bordeaux (2016).

[7] M.R. Bush, D.C. Mayer, *3-class field towers of exact length 3*, Journal of Number Theory **147** (2015), 766 – 777, DOI 10.1016/j.jnt.2014.08.010.

[8] A. Azizi, A. Zekhnini, M. Taous, D.C. Mayer, *Principalization of 2-class groups of type  $(2, 2, 2)$  of biquadratic fields  $\mathbb{Q}(\sqrt{p_1 p_2 q}, \sqrt{-1})$* , International Journal of Number Theory **11** (2015), no.4, 1177 – 1216, DOI 10.1142/S1793042115500645.

[9] A. Azizi, M. Talbi, M. Talbi, A. Derhem, D.C. Mayer, *The group  $\text{Gal}(k_3^{(2)}|k)$  for  $k = \mathbb{Q}(\sqrt{-3}, \sqrt{d})$  of type  $(3, 3)$* , International Journal of Number Theory (2016), DOI 10.1142/S1793042116501207.